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Proton Life-Time Problem In Finite Grand Unified Theories*

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Abstract

We study proton decay in finite supersymmetric SU(5) grand unified theories. We find that the dimension-five operators due to color triplet higgsino induce too rapid a proton decay. This behaviour can be traced to the large Yukawa couplings to the first generation that are necessary for finiteness.

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Proton decays are predicted in many grand unified theories (GUTs) [1]. Experimentally no proton decays have been observed [2]. The stringent experimental bounds on proton decays can provide interesting constraints on GUTs [3–5]. It has been shown that in the minimal supersymmetric (SUSY) SU(5) model, a large region in parameter space can be ruled out from the consideration of proton decays [4,5]. In this paper we study proton decays in a class of finite SUSY GUTs, namely the finite SUSY SU(5) models. We show that models considered so far are ruled out by experimental bounds on the proton life-time.

There have been many studies of finite GUTs [6–10]. This is a class of interesting GUTs. It supports strongly the hope that the ultimate theory does not need infinite renormalization. In order to have a finite theory to all orders, the β functions for the gauge coupling and Yukawa couplings have to be zero to all orders. The requirement that the β function of the gauge coupling be zero greatly restricts the allowed matter representations in a theory once the gauge group is given. The β function of the Yukawa couplings being zero can put additional constraints on the theory. A list of possible finite theories are given in Ref. [10]. A particularly interesting class of theories are the ones based on the $SU(5)$ gauge group with supersymmetry. There are several solutions satisfying the requirement that the β function of the gauge coupling be zero. However, the requirement that the β function for the Yukawa couplings be zero further restricts the allowed solutions. If one requires that $SU(5)$ is broken by the Higgs mechanism to $SU(3)_C \times SU(2)_L \times U(1)_Y$ with three generations of matter fields, only one solution is allowed with 5, $\bar{5}$, 10, $\bar{10}$ and 24 chiral multiplets with multiplicities (4,7,3,0,1) [10]. This model contains one 24 (Σ) of Higgs for the SU(5) breaking, $4(5+\bar{5})$ (H_α , $\bar{H}_\alpha$) of Higgs some of which will be used for electroweak breaking and the remaining $3(\bar{5}+10)$ are identified with the three generation matter fields. With this content, the most general superpotential that may be written, consistent with renormalizibility, $SU(5)$ invariance and R-parity conservation is of the form

$$W = qTr\Sigma^3 + MTr\Sigma^2 + \lambda_{\alpha\beta}\bar{H}_\alpha \Sigma H_\beta + m_{\alpha\beta}\bar{H}_\alpha H_\beta + \frac{1}{2}g_{ij\alpha}10_i 10_j H_\alpha + \bar{g}_{ij\alpha}10_i \bar{5}_j \bar{H}_\alpha \ , \qquad (1)$$

The indices α, β, and i, j run from 1 to 4 and 1 to 3, respectively.

The requirement that the β functions for the Yukawa couplings are zero at the one-loop level implies,

$$\begin{aligned}
\Sigma: &\quad \tfrac{189}{5}q^2 = 10g^2 - \lambda_{\alpha\beta}\lambda^{\alpha\beta} , \\
\bar{H}_\alpha: &\quad \bar{g}_{ij\alpha}\bar{g}^{ij\beta} = \tfrac{6}{5}(g^2\delta_\alpha^\beta - \lambda_{\alpha\gamma}\lambda^{\beta\gamma}) , \\
\bar{5}_i: &\quad \bar{g}_{ki\alpha}\bar{g}^{kj\alpha} = \tfrac{6}{5}g^2\delta_i^j , \\
H_\alpha: &\quad g_{ij\alpha}g^{ij\beta} = \tfrac{8}{5}(g^2\delta_\alpha^\beta - \lambda_{\gamma\alpha}\lambda^{\gamma\beta}) , \\
10_i: &\quad 2g_{ik\alpha}g^{jk\alpha} + 3g_{ik\alpha}g^{jk\alpha} = \tfrac{36}{5}g^2\delta_i^j .
\end{aligned} \tag{2}$$

This set of equations constrains the allowed values for the Yukawa couplings. However it is not restrictive enough so that all fermion masses and Kobayashi-Maskawa (KM) mixing angles can be predicted. The requirement of all-loop finiteness may further reduce the parameters. In Ref. [9] imposing an additional $Z_7 \times Z_3$ symmetry, a unique solution to eq.(2) is found

$$\begin{aligned}
g_{111}^2 = g_{222}^2 = g_{333}^2 = \frac{8}{5}g^2 , \\
\bar{g}_{111}^2 = \bar{g}_{222}^2 = \bar{g}_{333}^2 = \frac{6}{5}g^2 , \\
\lambda_{44} = g^2 , \qquad q^2 = \frac{5}{21}g^2 .
\end{aligned} \tag{3}$$

All other tri-linear couplings are zero. This is a very interesting theory because it is an all-loop finite theory [9]. All the Yukawa couplings are expressed in terms of gauge coupling g. This allows one to further predict some fermion masses.

In the above model only $H_4(\bar{H}_4)$ can develop vacuum expectation values in order that the doublet-triplet mass splitting is possible for the doublets which break $SU(3)_C \times SU(2)_L \times U(1)_Y$ to $U(1)_{em}$. Since the quadratic coupling $m_{\alpha\beta}$ is diagonal due to the $Z_7 \times Z_3$ discrete symmetry and $H_4(\bar{H}_4)$ do not couple to quarks and leptons, the fermions are all massless. This problem can be solved by softly breaking the $Z_7 \times Z_3$ discrete symmetry with off-diagonal $m_{\alpha\beta}$ entries. In this way one can find solutions such that each Higgs doublet can develop a vacuum expectation value and at the same time it is still possible to maintain the

doublet-triplet mass splitting. All fermions can now have masses [8]. Below the unification scale the model is effectively the same as the minimal SUSY standard model. Adding soft breaking terms, one can get supersymmetry breaking. Since the theory is spontaneously broken, the finiteness conditions do not restrict its renormalization properties. Carrying out the renormalization group analysis, the top quark mass has an upper-bound of 190 GeV. If the Higgs doublets which develop VEV only couple to the third generation, the top quark is determined to be between 175 to 190 GeV [9].

There are, however, several problems with this model. Because the Yukawa couplings are diagonal, all KM angles are zero. This is inconsistent with experiments. This problem can be solved by abandoning the diagonal solution to eq.(2). It is possible to find a solution of eq.(2) such that KM matrix can be reproduced. A possible solution is

$$\bar{g}_{ij\alpha} = \sqrt{\frac{6}{5}} g(\delta_{i,1}\delta_{\alpha,1} + \delta_{i,2}\delta_{\alpha,2} + \delta_{i,3}\delta_{\alpha,3})V_{ij} \tag{4}$$

with all other couplings the same as in eq.(3). Here V_{ij} is the KM matrix. This model has the same predictions for the quark masses. It does not satisfy the discrete $Z_7 \times Z_3$ symmetry.

There is another problem related to fermion masses in this model, as has been noted in Ref. [7]. Because there are only 5 and $\bar{5}$ Higgs representations to generate masses for quarks and charged leptons, this model also predicts the wrong mass relations for the first two generations: $m_e = m_d$, $m_\mu = m_s$ at the GUT scale. This is a common problem for SU(5) models with only 5 and $\bar{5}$ Higgs representations to generate fermion masses. If higher dimension operators are somehow allowed, this problem can be solved. For example, adding a $(10 \times \bar{5})(\Sigma \bar{H}_\alpha)$ term can correct the wrong mass relations. However, this solution is not consistent with the finiteness conditions. This, however, is not the major problem. In the following we will show that even if we relax the conditions to allow the above additions to the theory, the model has another problem. It predicts too rapid a proton decay.

There are several mechanisms by which proton decays may be induced in SUSY SU(5) theories. The exchanges of heavy gauge bosons is one. In the finite theory discussed here the contributions from heavy gauge bosons are the same as in the minimal SUSY SU(5).

The proton decays due to this mechanism have been extensively studied [3,5,11], and can easily satisfy the experimental lower bounds [5,11]. In the minimal SUSY SU(5) model, exchange of scalar color triplets will also generate dimension-six operators which can mediate proton decays. There, however, due to small Yukawa couplings, the decay rates due to this mechanism is much smaller than the contribution from the heavy gauge bosons. In the finite SUSY SU(5), the Yukawa couplings are of the same order of magnitude as the gauge coupling. The scalar color triplets induced proton decays are comparable with the heavy gauge boson contributions, and can easily satisfy the experimental bound because the scalar color triptlet masses are also of the same order of magnitude as the gauge bosons and could even be somewhat heavier. The most significant contributions to the proton decays come from the dimension-five operator induced by exchanging color triplet higgsinos H_C and $\bar{H}_C$ of H_α and $\bar{H}_\alpha$ [3–5]. In the minimal SUSY SU(5) model, this mechanism is the dominant one and considerably restricts the allowed region in parameter space of the model [5]. In the finite SU(5) model, experimental bounds on proton decays all but make these models unacceptable.

The diagrams for the dimension-five induced four-fermion operator responsible for proton decays are shown in Fig. 1. There are other similar contributions that arise by replacing the chargino $\tilde{w}$ by a gluino, or a zino. It has been argued that the dominant ones are from chargino exchange [3–5], and we shall only need to consider chargino constributions. The four-fermion baryon number violating effective Lagrangian at 1 GeV can be written down explicitly as [5]

$$\begin{aligned} L = & \frac{\alpha_2}{2\pi M_{H_C^\alpha}} g_{ii\alpha}\bar{g}_{kk\alpha}V^*_{jk}A_S A_L \\ & \times[(u_i d'_i)(d'_j\nu_k)(f(u_j,e_k)+f(u_i,d'_i)) + (d'_i u_i)(u_j e_k)(f(u_i,d_i)+f(d'_j,\nu_k)) \\ & +(d'_i\nu_k)(d'_i u_j)(f(u_i,e_k)+f(u_i,d'_j)) + (u_i d'_j)(u_i e_k)(f(d'_i,u_j)+f(d'_i,\nu_k))] \ , \end{aligned} \tag{5}$$

where $d'_i = V_{il}d_l$; $f(a,b) = m_{\tilde{w}}[m^2_{\tilde{a}}\ln(m^2_{\tilde{a}}/m^2_{\tilde{w}})/(m^2_{\tilde{a}}-m^2_{\tilde{w}}) - (m_{\tilde{a}}\to m_{\tilde{b}})]/(m^2_{\tilde{a}}-m^2_{\tilde{b}})$ is from the loop integral, and $m_{\tilde{a},\tilde{b}}$ are the s-fermion masses, $A_S \approx 0.59$, $A_L \approx 0.22$ [5] are the QCD correction factors for the running from M_{GUT} to SUSY breaking scale and from SUSY

breaking scale to 1 GeV, respectively, and the Yukawa couplings are evaluated at 1 GeV.

Because all $g_{ii\alpha}$ and $\bar{g}_{jj\alpha}$ are equal in the model we are considering, the dominant contributions to the proton decays will be the ones involving only particles in the first generation. The dominant baryon number violating decay modes are: $p \to \pi^+ \bar{\nu}_e$, $p \to \pi^0(\eta) e^+$, $n \to \pi^0(\eta)\bar{\nu}_e$, $p \to \pi^- e^+$.

Finally to obtain the life times of the proton and neutron, we employ the chiral Lagrangain approach to parametrize the hadronic matrix elements [12]. We have

$$
\begin{aligned}
\Gamma(p \to \pi^+ \bar{\nu}_e) &= 2\Gamma(n \to \pi^0 \bar{\nu}_e) = \beta^2 \frac{m_N}{32\pi f_\pi^2} |C(duu\nu_e)(1+D+F)|^2 \ , \\
\Gamma(n \to \eta \bar{\nu}_e) &= \beta^2 \frac{(m_N^2 - m_\eta^2)^2}{64\pi f_\pi^2 m_N^3} 3|C(duu\nu_e)(1 - \frac{1}{3}(D-3F))|^2 \ , \\
\Gamma(n \to \pi^- e^+) &= 2\Gamma(p \to \pi^0 e^+) = \beta^2 \frac{m_N}{32\pi f_\pi^2} |C(duue)(1+D+F)|^2 \ , \\
\Gamma(p \to \eta e^+) &= \beta^2 \frac{(m_N^2 - m_\eta^2)^2}{64\pi f_\pi^2 m_N^3} 3|C(duue)(1 - \frac{1}{3}(D-3F))|^2 \ ,
\end{aligned}
\tag{6}
$$

where $D = 0.81$ and $F = 0.44$, which arise from the strong interacting baryon-meson chiral Lagrangian, $f_\pi = 132$ MeV is the pion decay constant, and m_N and m_η are the neucleon and η meson masses, respectively. The parameter β is estimated to be in the range 0.03 GeV^3 to 0.0056 GeV^3 [13]. The parameters $C(duu\nu)$ and $C(duue)$ are the coefficients of the operators $(du)(u\nu)$ and $(du)(ue)$ which can be read off from eq.(5). We have

$$
\begin{aligned}
C(duu\nu_e) &= \frac{4\alpha_{em}^2}{\sin^4\theta_W} \frac{\bar{m}_b \bar{m}_t}{m_W^2 \sin 2\beta_H} \frac{A_S A_L}{M_{H_C^1}} V_{ud}^2 V_{ud}^* (f(u,e) + f(u,d)) \ , \\
C(duue) &= \frac{4\alpha_{em}^2}{\sin^4\theta_W} \frac{\bar{m}_b \bar{m}_t}{m_W^2 \sin 2\beta_H} \frac{A_S A_L}{M_{H_C^1}} V_{ud} V_{ud}^* (f(u,e) + f(u,d)) \ .
\end{aligned}
\tag{7}
$$

In the above we have used $g_{111}\bar{g}_{111} = g_2^2 \bar{m}_b \bar{m}_t / m_W^2 \sin 2\beta_H$ as a good approximation. Here the quark masses are at 1 GeV. $\tan\beta_H$ is the ratio of the vacuum expectation value of H_1 to that of $\bar{H}_1$. It is predicted to be about 50. The top quark mass at 1 GeV $\bar{m}_t$ is about 470 GeV [9]. Using these values, we obtain the partial life-times for some of the baryon number violating decays as

$$
\tau\ (\ p \to \pi^0 e^+) \approx \tau(n \to \pi^0 \bar{\nu}_e) \approx 6 \times 10^{17} \times P \ \ years \ ,
$$

$$\tau(p\rightarrow \pi^+\bar{\nu}_e) \approx \tau(n \rightarrow \pi^- e^+) \approx 3 \times 10^{17} \times P \ \ years \ , \tag{8}$$

$$\tau \ (\ p \rightarrow \eta e^+) \approx \tau(n \rightarrow \eta\bar{\nu}_e) \approx 2 \times 10^{18} \times P \ \ years \ ,$$

where

$$P = \left(\frac{0.003 \ GeV^3}{\beta}\right)^2 \left(\frac{M_{H_C}}{10^{17} \ GeV}\frac{TeV^{-1}}{f(u,d)+f(u,e)}\right)^2 \ . \tag{9}$$

The value $\beta = 0.003 GeV^3$ is at the lower end of the estimations. The color triplet higgsino mass can not be too much larger than 10^{17} GeV. Even if we allow it to be the same order as the Planck mass, these partial life-times are in contradiction with experiments if the factor $I = TeV^{-1}/(f(u,d)+f(u,e))$ is of order one. In the model discussed above, there are no other possible sources to cancel the above contributions. The only possible way out is to have a very small I. If all s-fermion and chargino masses are of order TeV, the factor I has to be of O(1). If chargino is much heavier than s-fermions, $f \approx (\ln(m^2_{\tilde{w}}/m^2_{\tilde{f}}))/m_{\tilde{w}}$. In order to satisfy the experimental bounds on the partial life-times, the mass of the chargino has to be in the $10^8 \ TeV$ region for $m_{H_C} = 10^{17}$ GeV. If the s-fermion masses are much larger than the chargino mass, $f \approx m_{\tilde{w}}/m^2_{\tilde{f}}$. In this case, the s-fermion masses have to be larger than 2×10^3 TeV for $m_{\tilde{w}} > 100$ GeV and $m_{H_C} = 10^{17}$ GeV. All these solutions require that SUSY be broken at a scale much much larger than a few TeV. However such solutions spoil the nice feature of solving the hierarchy problem that is the rationale for using SUSY theories in the first place. This high scale may also cause problem for the correct predictions of $\sin^2\theta_W$. From these considerations, the model discussed above is either ruled out, or quite unattractive needing a large SUSY breaking scale.

In order to solve the proton life-time problem, one needs to find solutions to the finiteness conditions such that the Yukawa couplings for the first generation are much smaller than the gauge coupling. This is not ruled out but it may be difficult to find such an all-loop finite theory. We expect this problem to arise in most finite thoeries of grand unification that allow proton decay.

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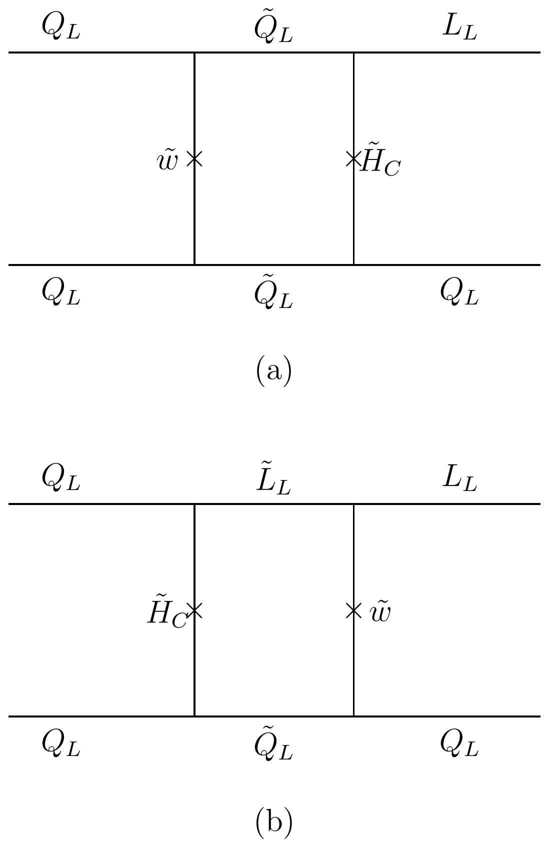

Fig. 1. Dimenison-five proton decay operator due to color triplet higgsino exchange.